2 Logical Equivalences and Selected Forms of Statements

8. (a) Express the compound statement $A \lor B$ using only the basic connectives \neg and \land .

(b) Suppose we have the table on the right. Find C.

9. The following compound statement is given:

$$(\neg p \land \neg q) \lor \neg (q \land r)$$

Construct a truth table.

10. Is the following reasoning correct?

- I think, therefore I am. I think, therefore I reason. Conclusion: I am, therefore I reason.
- 11. (exam, November 2021) The following compound statement is given:

$$(A \Rightarrow B) \land (B \Rightarrow C).$$

- (a) Construct a truth table.
- (b) What can be said about the truth of statements B and C if the compound statement is true and A is a true statement?
- (c) Write the **negation** of the given compound statement in disjunctive form.

12. The following compound statement is given:

$$(\neg A \lor B) \Leftrightarrow (C \Rightarrow A).$$

- (a) Construct a truth table.
- (b) What can be said about the truth of statements A and C if the compound statement is false and B is true?
- (c) Write the negation of the given compound statement in disjunctive form.

13. For the given compound statement, construct a truth table and determine its conjunctive and disjunctive normal forms:

$$(A \lor B) \land (\neg A \lor C) \Leftrightarrow \neg (B \Rightarrow \neg C).$$

14. For the following compound statement, provide the truth table, and determine the conjunctive and disjunctive normal forms:

$$(A \Rightarrow (B \Rightarrow C)) \Rightarrow ((A \Rightarrow B) \Rightarrow (A \Rightarrow C))$$

15. The following compound statement is given:

$$\neg A \land (A \Leftrightarrow B) \Rightarrow \neg B. \tag{1}$$

What can be said about the truth of implication (1)? In other words, is the statement true for all assignments, or is there an assignment for which the statement is false? Provide the solution in two different ways:

p	q	C
0	0	1
0	1	1
1	0	0
1	1	1

- (i) Using a truth table.
- (ii) Without using a truth table.

16. Determine the compound statement ${\cal I}$ such that

$$(A \Rightarrow (\mathcal{I} \Rightarrow \neg B)) \Rightarrow (A \land B) \lor \mathcal{I}$$

is a tautology.

All above math problems are taken from the following website: https://osebje.famnit.upr.si/~penjic/teaching.html. THE READER CAN FIND ALL SOLUTIONS TO THE GIVEN PROBLEMS ON THE SAME PAGE.